Attenuation and Amplification of Acoustic Waves in Nondegenerate Semiconductors in the Presence of dc Fields*

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(Received 16 November 1970)

We have solved the Boltzmann equation for the electrons interacting with an acoustic wave in the presence of a moderate dc electric field to obtain the expressions for conductivity tensors and hence the absorption coefficient. In contrast with earlier work, the energy dependence of relaxation time as given by acoustic-phonon scattering has been taken into account. Calculations show that for $ql<1$ the absorption changes its sign when the ratio of average drift velocity to sound velocity is equal to or greater than unity. For $ql>1$, however, the threshold drift velocity required for amplification is a function of ω/A, and for most of the realms of ω/A of interest, it is between $\frac{2}{3}$ and $\frac{1}{2}$ of sound velocity.

I. INTRODUCTION

In the last decade much theoretical work[1–6] has been done on the attenuation and amplification of acoustic waves in semiconductors. Most of these analyses assume a constant relaxation time for electron collisions. However, recently Jacoboni and Prohofsky[7] have realized the importance of considering an energy-dependent relaxation time in the investigation of the attenuation of acoustic waves in nondegenerate piezoelectric semiconductors. Using kinetic-theory techniques, they have derived the expression for the absorption coefficient for the two cases of scattering in which acoustic-phonon scattering or ionized-impurity scattering is the dominant scattering mechanism.

In the present paper we have analytically investigated the propagation of an acoustic wave in a nondegenerate n-type semiconductor in the presence of a moderate external dc field. The well-known model of a free-electron gas developed by Cohen, Harrison, and Harrison[8] has been used to take into account the sound-wave–electron coupling due to the deformation potential. Our analysis considers acoustic-phonon scattering as the dominant electron-scattering mechanism. This restricts our analysis to the temperature range where acoustic-phonon scattering is the dominant scattering mechanism (i.e., 77–300 °K). Following Spector,[9] we solve the linearized Boltzmann equation to obtain an expression for the anisotropic part of the distribution function of electron velocities. This is further used to obtain the expressions for the conductivity tensors and hence the absorption coefficient in the two ranges of frequencies $ql>1$ and $ql<1$.

It is seen from our calculations that, for $ql<1$, the threshold drift velocity required by the electrons for amplification is equal to the velocity of sound, while for $ql>1$ it is a function of ω/A (where ω is the frequency of the wave and A is the collision frequency of the electrons), but for all the physical values of ω/A it is between $\frac{2}{3}$ and $\frac{1}{2}$ of the sound velocity.

II. BOLTZMANN TRANSFER EQUATION

The linearized Boltzmann transfer equation for electrons in the presence of an acoustic wave and a longitudinal dc field (along the direction of propagation, i.e., z axis) may be written as[5]

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}\,'}-\frac{e}{m}\left(\vec{\epsilon}+\frac{\vec{q}\vec{q}\cdot\overleftrightarrow{C}\cdot\vec{u}}{ei\omega}+\vec{E}_d\right)\cdot\frac{\partial f}{\partial \vec{v}}=-\nu\left[f-f^0+\frac{\partial f^0}{\partial E}\left(m\vec{u}\cdot\vec{v}+\frac{N_1}{N_0}kT\right)\right], \tag{1}$$

where f is the distribution function of electron velocities, f^0 is the unperturbed distribution of electron velocities and has been assumed to be Maxwellian, $\vec{r}\,'$ is the position vector, $\vec{q}$ is the wave vector of the sound wave, t is the time, $-e$ is the electronic charge, m is the effective mass of the electron, $\vec{\epsilon}$ is the electric vector associated with the acoustic wave, k is Boltzmann's constant, T is the temperature, E is the electric energy, $\vec{E}_d$ is the external dc field, $\vec{v}$ is the velocity, ω is the frequency of the acoustic wave, N_0 is the electron concentration, N_1 is the fluctuating part of the electron concentration which varies as $e^{i(\omega t-\vec{k}\cdot\vec{r}\,')}$, $\vec{u}$ is the velocity field, $\overleftrightarrow{C}$ is the deformation potential of the semiconductor, and ν is the collision frequency, which is given by[10]

$$\nu=v/l=Ax^{1/2}, \quad x=mv^2/2kT. \tag{2}$$

Expanding the distribution function in isotropic and anisotropic parts, we have

$$f=f^0+f'\ . \tag{3}$$

Now we can further separate $f'_z(v_z)$ into time-independent and time-dependent parts as follows:

$$f_z' = f_0' + f_1'(t) \quad . \tag{4}$$

f_0' in Eq. (4) is time independent, while f_1' is the time-dependent part, which varies as $e^{i(kz-\omega t)}$. Substituting Eqs. (3) and (4) in the Boltzmann equation and equating the terms varying as $e^{i(kz-\omega t)}$ on both sides, one obtains the following solution, correct to first order in the electric field, for f_1':

$$f_1' = \frac{e\vec{\epsilon}_s\cdot\vec{v}}{avp}\frac{\partial f_0}{\partial E} - \frac{kT}{p}\frac{N_1}{N_0}\frac{\partial f_0}{\partial E} + \frac{e^2}{mpav}\left[\frac{\vec{E}_d\cdot\vec{\epsilon}_s}{av}\frac{\partial f_0}{\partial E} - \frac{\vec{E}_d\cdot\vec{v}}{av^3}\vec{\epsilon}_s\cdot\vec{v}\frac{\partial f_0}{\partial E} + \frac{\vec{E}_d\cdot\vec{v}\vec{\epsilon}_s\cdot\vec{v}}{av}\frac{\partial^2 f_0}{\partial E^2}\right]$$
$$+ \frac{e}{avpm}\left[\frac{e}{avp}\left(\vec{E}_d\cdot\vec{v}\vec{\epsilon}_s\cdot\vec{v}m\frac{\partial^2 f_0}{\partial E^2} + \frac{\partial f_0}{\partial E}\vec{E}_d\cdot\vec{\epsilon}_s\right) - \frac{e\vec{\epsilon}_s\cdot\vec{v}}{a^2v^2p^2}\frac{\partial f_0}{\partial E}\left(\frac{\vec{E}_d\cdot\vec{v}A}{v} + i\vec{q}\cdot\vec{E}_d\right) - \frac{kT}{p}\frac{N_1}{N_0}\frac{\partial^2 f_0}{\partial E^2}m\vec{v}\cdot\vec{E}_d - i(\vec{q}\cdot\vec{v}-\omega)\frac{kT}{p^2}\right.$$
$$\left.\times\frac{N_1}{N_0}\frac{\partial f_0}{\partial E}\frac{\vec{E}_d\cdot\vec{v}}{av^3} + \frac{i\vec{q}\cdot\vec{E}_d}{avp^2}kT\frac{N_1}{N_0}\frac{\partial f_0}{\partial E}\right] \quad , \tag{5}$$

where

$$\vec{v}_d = e\,\vec{E}_d/m\nu \,,$$

$$p = 1 - i\omega/\nu + (i\vec{q}\cdot\vec{v})/v \,,$$

$$a = \nu/v = l^{-1} \,,$$

$$\vec{\epsilon}_s = \vec{\epsilon} + (\vec{q}\vec{q}\cdot\overleftrightarrow{C}\cdot\vec{u})/ei\omega \,.$$

Substituting expression for f_1' in the general equation for the electric current,

$$J_z = -e\int d\vec{v}\,v_z f, \tag{6a}$$

one gets the following expression for the electronic current induced by the sound wave:

$$J = \overleftrightarrow{\sigma}\cdot\left(\vec{\epsilon} + \frac{\vec{q}\vec{q}\cdot\overleftrightarrow{C}\cdot\vec{u}}{ei\omega}\right) - \vec{R}\,N_1 e v_s + \overleftrightarrow{\Sigma}\cdot\left(\vec{\epsilon} + \frac{\vec{q}\vec{q}\cdot\overleftrightarrow{C}\cdot\vec{u}}{ei\omega}\right) \quad , \tag{6b}$$

where

$$\overleftrightarrow{\sigma} = \frac{e^2}{a}\int d^3v\left[\left(-\frac{\partial f_0}{\partial E}\right)\frac{\vec{v}\vec{v}}{vp} + \frac{e\vec{E}_d\cdot\vec{v}}{amv^2p^2}\left(-\frac{\partial f_0}{\partial E}\right) - \frac{e\vec{E}_d\cdot\vec{v}\vec{v}\vec{v}}{av^2p^2}\frac{\partial^2 f_0}{\partial E^2} - \frac{e}{mav^3p^3}\left(\frac{\vec{E}_d\cdot\vec{v}\vec{v}\vec{v}}{v} + \frac{i\vec{E}_d\cdot\vec{q}\vec{v}\vec{v}}{a}\right)\left(-\frac{\partial f_0}{\partial E}\right)\right] , \tag{7}$$

$$\vec{R} = \frac{kT}{N_0 v_s}\int d^3v\left[\frac{\vec{v}}{p}\left(-\frac{\partial f_0}{\partial E}\right) - \frac{e\vec{E}_d\cdot\vec{v}\vec{v}}{avp^2}\frac{\partial^2 f_0}{\partial E^2} + \frac{ie\vec{E}_d\cdot\vec{v}\vec{v}\vec{v}}{ma^2v^4p^3}(\vec{q}\cdot\vec{v}-\omega)\left(-\frac{\partial f_0}{\partial E}\right) - \frac{ie\vec{E}_d\cdot\vec{q}\vec{v}}{ma^2v^2p^3}\left(-\frac{\partial f_0}{\partial E}\right)\right] , \tag{8}$$

$$\overleftrightarrow{\Sigma} = \frac{e^2}{a}\int d^3v\left[\frac{e}{mpv}\left(\frac{\vec{E}_d\cdot\vec{v}}{av} - \frac{\vec{E}_d\cdot\vec{v}\vec{v}\vec{v}}{av^3}\right)\left(-\frac{\partial f_0}{\partial E}\right) - \frac{e\vec{E}_d\cdot\vec{v}\vec{v}\vec{v}}{pav^2}\frac{\partial^2 f_0}{\partial E^2}\right] \quad . \tag{9}$$

Following Spector,[9] the absorption coefficient of the acoustic wave is given by

$$\frac{\alpha}{N_0 m\nu/\rho v_s} = \frac{(1 + B_{zz} - C_{zz}q_z^2 B_{zz}/im\omega\nu)[1 - \sigma_{zz}' - iql(\omega/A)x^{-1}(C_{zz}/mv_s^2)(\sigma_{zz}' + \Sigma_{zz}')]}{\sigma_{zz}' + \Sigma_{zz}' - i\omega\nu/\omega_p^2} \quad , \tag{10}$$

where

$$B_{zz} = -i\omega_p^2/\omega\nu \,, \tag{11a}$$

and σ_{zz}' and Σ_{zz}' are the components of the tensor defined by equations

$$\overleftrightarrow{\sigma}' = (1 - \overleftrightarrow{R})^{-1}\overleftrightarrow{\sigma}/\sigma_0, \tag{11b}$$

$$\overleftrightarrow{\Sigma}' = (1 - \overleftrightarrow{R})^{-1}\overleftrightarrow{\Sigma}/\sigma_0, \tag{11c}$$

$$\omega_p^2 = 4\pi N_0 e^2/m \,, \tag{11d}$$

where

$$\sigma_0 = N_0 e\mu_L = 4\pi N_0 e^2/3(\sqrt{\pi})mA$$

is the dc conductivity, μ_L is the mobility for the dominant acoustic-phonon scattering,[11] ρ is the density of the semiconductor, C_{zz} is the deformation potential, and $\overleftrightarrow{R}$ is a tensor defined by Spector.[9]

III. CALCULATION OF CONDUCTIVITY TENSORS AND ABSORPTION COEFFICIENT

In this section, we shall compute the conductivity tensors $\overleftrightarrow{\sigma}$, $\overleftrightarrow{\Sigma}$, and $\vec{R}$ by evaluating the integrals involved in Eqs. (7)–(9). While integrating over the volume, it is convenient to first integrate over ϕ, then over v, and finally over θ. Thus, after integrating over ϕ and v, one obtains

$$\delta_{zz}=\frac{2\pi e^2C_nv_0^4}{kTa}\left[\frac{iA}{\omega}\int_{-1}^{+1}F_2rt^2dt+\frac{a_d}{v_0}\frac{A}{\omega^2}\int_{-1}^{+1}(3F_1-2F_2)\,r^2t\,dt+\frac{gla_d}{v_0}\frac{A^2}{\omega^3}\int(6F_1-9F_2+2F_3)r^3t^2dt\right.$$

$$\left.-\frac{2a_d}{v_0}\frac{A}{\omega^2}\int(5F_2-2F_3)r^2t^3\,dt-\frac{iA^2}{\omega^3}\frac{a_d}{v_0}\int_{-1}^{+1}(6F_1-9F_2+2F_3)r^3t^3dt\right], \tag{12a}$$

$$R_z=\frac{2\pi C_n v_0^4}{N_0v_s}\left[\frac{iA}{\omega}\int_{-1}^{+1}F_2rt\,dt-\frac{2a_d}{v_0}\frac{A}{\omega^2}\int_{-1}^{+1}(5F_2-2F_3)r^2t^2\,dt-ql\frac{a_d}{v_0}\frac{A^2}{\omega^3}\int_{-1}^{+1}(6F_1-9F_2+2F_3)\,r^3t^3\,dt\right.$$

$$\left.+ql\frac{a_d}{v_0}\frac{A^2}{\omega^3}\int_{-1}^{+1}(6F_1-9F_2+2F_3)r^3\,t\,dt\right], \tag{12b}$$

$$\Sigma_{zz}=\frac{2\pi e^2C_nv_0^4}{akT}\left[-i\frac{a_d}{v_0\omega}\int_{-1}^{+1}F_1rt\,dt+\frac{ia_d}{v_0\omega}\int_{-1}^{+1}F_1rt^3dt+\frac{2ia_d}{v_0\omega}\int_{-1}^{+1}F_2rt^3dt\right], \tag{12c}$$

where

$$C_n=\frac{N_0}{\pi^{3/2}v_0^3}$$

is the normalization constant,

$$r(t)=\frac{v_s/v_0}{i/ql-t},\quad a_d=eE_d/m\ ,$$

$$t=\cos\theta,\qquad v_0=(2kT/m)^{1/2}\ ;$$

$$F_1=\tfrac{1}{2}-\tfrac{1}{2}r\sqrt{\pi}+\tfrac{1}{2}r^2e^{-r^2}E_1(-r^2)+\tfrac{1}{2}i\pi r^2W(-r), \tag{13a}$$

$$F_2=\tfrac{1}{2}-\tfrac{1}{4}r\sqrt{\pi}+\tfrac{1}{2}r^2-\tfrac{1}{2}r^3\sqrt{\pi}+\tfrac{1}{2}r^4e^{-r^2}E_1(r^2)+\tfrac{1}{2}i\pi r^4W(-r), \tag{13b}$$

$$F_3=1-\tfrac{3}{8}r\sqrt{\pi}+\tfrac{1}{2}r^2-\tfrac{1}{4}r^3\sqrt{\pi}+\tfrac{1}{2}r^4-\tfrac{1}{2}r^5\sqrt{\pi}+\tfrac{1}{2}r^6e^{-r^2}E_1(-r^2)+\tfrac{1}{2}\pi r^6iW(-r)\ ; \tag{13c}$$

$$E_1(z)=\int_z^{\infty}\frac{e^{-t}}{t}\,dt, \tag{13d}$$

$$W(z)=\frac{2iz}{\pi}\int_0^{\infty}\frac{e^{-t^2}}{z^2-t^2}\,dt\quad(\mathrm{Im}\,z>0) \tag{13e}$$

are the well-known tabulated integrals.[12]

Jacoboni and Prohofsky[7] have shown that it is a very good approximation for all frequencies to take only the first three terms in Eq. (13) to evaluate $\overleftrightarrow{\sigma}$, $\overleftrightarrow{\Sigma}$, and $\vec{R}$. Thus, one obtains, after integrating over θ, the following expressions in the two ranges of frequencies $ql<1$ and $ql>1$:

Case 1: $ql<1$

$$\sigma_{zz}/\sigma_0=1+i(\omega/2A)\sqrt{\pi}+3(\omega/A)\mu i, \tag{14a}$$

$$\Sigma_{zz}/\sigma_0=+\mu(\omega/A)i\ , \tag{14b}$$

$$R_z=-\frac{2}{3\sqrt{\pi}}\frac{q^2l^2i}{\omega/A}+\frac{4}{3\sqrt{\pi}}\mu\ , \tag{14c}$$

where

$$\mu=a_d/Av_s.$$

Case 2: $ql>1$

$$\frac{\sigma_{zz}}{\sigma_0}=\frac{3}{q^2l^2}\left(1-i\frac{\omega}{2A}\sqrt{\pi}\right)+\frac{3\pi}{2q^3l^3}\left(-1+i\frac{\omega}{A}\sqrt{\pi}+\frac{\omega^2}{A^2}\right), \tag{15a}$$

$$\frac{\Sigma_{zz}}{\sigma_0}=-\frac{3}{2}\pi\mu\frac{\omega}{A}\frac{1}{q^3l^3}\left(i+(\sqrt{\pi})\frac{\omega}{A}\right), \tag{15b}$$

$$R_z=-\frac{2i}{(\omega/A)\sqrt{\pi}}+\frac{i\sqrt{\pi}}{(\omega/A)ql}+\frac{\pi}{2ql}\ . \tag{15c}$$

Now, one can obtain the expression for the absorption coefficient α by substituting (14) and (15) into Eq. (10). Thus, we have

$$\frac{\alpha}{N_0mv/\rho v_s}=\frac{\omega^2}{\omega_p^2}\,\frac{\dfrac{4}{3\sqrt{\pi}}x^{-1/2}\dfrac{q^2l^2}{(\omega/A)^2}\dfrac{C_{zz}}{mv_s^2}+\left(\dfrac{C_{zz}}{mv_s^2}\right)^2x^{-1}\left(1-\dfrac{\langle v_d\rangle}{v_s}\right)+\dfrac{4}{9\pi}\dfrac{q^4l^4}{\omega^4/A^4}}{\dfrac{1}{(\omega/A)^2}\left(1-\dfrac{\langle v_d\rangle}{v_s}\right)^2+\dfrac{4}{9\pi}\dfrac{q^4l^4}{\omega^4/A^4}\dfrac{\omega^2}{\omega_p^2}}\qquad\text{for } ql<1, \tag{16}$$

where $\langle v_d\rangle=\mu_LE_d=(4e/3mA\sqrt{\pi})E_d$ is the average drift velocity of the carriers, and

$$\frac{\alpha}{N_0 m\nu/\rho v_s} = \frac{\frac{3}{2}(C_{zz}/mv_s^2)^2 x^{-1/2}(\omega^2/A)\pi}{q^3 l^3} \frac{(\omega^2/A^2+1) - \frac{3}{4}(\langle v_d\rangle/v_s)[(\omega^2/A^2)\pi+2]}{4[(\omega^2/A^2)\pi]^{-1}+1} \quad \text{for } ql > 1. \tag{17}$$

In obtaining the above expressions for α we have assumed[5] that $C_{zz}/mv_s^2 \gg 1$, a condition which is always satisfied in nondegenerate semiconductors. We have also neglected the terms which are an order of magnitude smaller than those retained.

IV. DISCUSSION

Equations (16) and (17) give the expressions for the absorption coefficient α for $ql < 1$ and $ql > 1$, respectively. It is seen from these equations that the absorption changes its sign when

$$\langle v_d\rangle/v_s \geq 1 \quad \text{for } ql < 1 \text{ (when } C_{zz}/mv_s^2 > v_s^2/v_0^2), \tag{18}$$

and

$$\frac{\langle v_d\rangle}{v_s} \geq \frac{4}{3}\frac{\omega^2/A^2+1}{(\omega^2/A^2)\pi+2} \quad \text{for } ql > 1. \tag{19}$$

Thus, for $ql < 1$ the threshold drift velocity of the electrons that changes absorption into amplification is equal to the velocity of sound. This is in agreement with the result[9] that is obtained for a degenerate system with an electron-velocity-independent collision frequency. However, for $ql > 1$ a more interesting result is obtained. In this case (i.e., $ql > 1$), the threshold drift velocity is a function of ω/A, i.e., frequency of the wave and the crystal temperature [see Eq. (19)]. In this case (i.e., $ql > 1$) the value of ω/A lies between 0.1 and 1. Taking $\omega/A = 0.1$ and 1, we obtain from (19) the following condition:

$$\langle v_d\rangle/v_s \gtrsim \tfrac{4}{3}\tfrac{1.01}{2.03} \approx \tfrac{2}{3} \quad (\text{for } \omega/A = 0.1),$$

$$\langle v_d\rangle/v_s \gtrsim \tfrac{8}{15.4} \approx \tfrac{1}{2} \quad (\text{for } \omega/A = 1).$$

Thus for $ql > 1$, $\langle v_d\rangle/v_s$ lies between $\frac{1}{2}$ and $\frac{2}{3}$.

Next, we shall compare our results with those obtained by Spector[4] (he assumes nondegenerate statistics but constant relaxation time). From Eqs. (14a)–(14c) and from (15a)–(15c), one can obtain the expressions for $(\sigma'_{zz} + \Sigma'_{zz})\sigma_0$ in the following form for $ql \ll 1$ and $ql \gg 1$:

$$(\sigma'_{zz} + \Sigma'_{zz})\sigma_0 \simeq \frac{\sigma_0}{1 - (\langle \vec{v}_d\rangle \cdot \vec{q})/\omega + (2/3\sqrt{\pi})(q^2 l^2)(\omega/A)^{-1} i} \quad \text{for } ql \ll 1 \tag{20}$$

and

$$(\sigma'_{zz} + \Sigma'_{zz})\sigma_0 \simeq -i\omega\frac{q_d^2}{q^2}\left[1 + i(\sqrt{\pi})\left(\frac{\omega}{qv_0} - \frac{1}{2}\pi\frac{\langle \vec{v}_d\rangle\cdot\vec{q}}{qv_0}\right)\right] \quad \text{for } ql \gg 1, \tag{21}$$

where q_d is as defined by Spector.[4]

These expressions are of the same form as the Eqs. (2.13) and (2.14) of Spector,[4] except for different factors appearing in various terms of these equations. These extra factors are obviously due to the electron-velocity-dependent collision frequency. It is interesting to note that in the absence of the dc field our expressions (14a) and (14c) for σ_{zz} and R_z are identically reduced to those of Jacoboni and Prohofsky[7] [their Eqs. (20) and (21)]. It remains to be mentioned that our analysis is valid when the use of the linearized Boltzmann equation is justified. The theory also becomes applicable to a p-type semiconductor if $-e$ and m are replaced by $+e$ and m_h, respectively, in the final result of the analysis.

ACKNOWLEDGMENTS

I am thankful to Professor B. V. Paranjape for making various suggestions. My sincere thanks are also due Dr. David Thornton for a number of helpful discussions. I am thankful to the Institute of Theoretical Physics for a fellowship.

*Work supported in part by the Defence Research Board of Canada.

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